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H.M. Fried
Brown University

B. Müller
Duke University

Y. Gabellini
Université de Nice

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THE SPIN DEPENDENCE OF SWIFT PROTON COLLISIONS

A.T. BATES, N.H. BUTTIMORE and K.P. BYRNE
School of Mathematics, University of Dublin
Trinity College, Dublin 2, Ireland
E-mail: Nigel.Buttimore@tcd.ie

The relation between elastic amplitudes for polarized proton-proton collisions and spin-dependent scattering observables is studied near the forward direction with a view to providing information on particular amplitudes at low values of momentum transfer. Such information is particularly important, for example, in considerations relating to the use of the transverse single-spin asymmetry in estimating the beam polarization achieved by the increasing number of accelerators aspiring to offer spin-polarized hadronic particles at higher energy.

1 Introduction

A study of the spin-dependent reaction parameters of the differential cross section in the low momentum transfer region would provide significant information on high energy spin dependence in hadronic elastic and diffractive collisions. Of particular interest is the Coulomb Nuclear Interference CNI region at small $-t$ for proton proton elastic scattering, for example, at high energy polarized hadron colliders such as the Relativistic Heavy Ion Collider (RHIC). In addition, longitudinal and transverse spin-polarized total cross section differences limit the rôle played by, at least the imaginary parts of, spin-inelastic particle-elastic collision amplitudes.

2 Elastic helicity amplitudes

Theoretical studies of nucleon-nucleon scattering indicate that there may be a spin-dependent element[11] that does not vanish with increasing energy.[12] In the s-channel helicity formalism[1] for pp elastic scattering we can write the transverse single-spin asymmetry as

$$A_N = -\frac{2\,\mathrm{Im}[\phi_5^*\,(\phi_1+\phi_2+\phi_3-\phi_4)]}{|\phi_1|^2+|\phi_2|^2+|\phi_3|^2+|\phi_4|^2+4|\phi_5|^2}\,. \tag{1}$$

Introduce an amplitude $\phi_+(s,t)$ for the spin-average of the two helicity nonflip amplitudes $\phi_1 = \langle + + |\phi| + +\rangle$ and $\phi_3 = \langle + - |\phi| + -\rangle$ with a corresponding difference amplitude $\phi_-(s,t)$

$$\phi_+ = \frac{\phi_1 + \phi_3}{2}, \qquad \phi_- = \frac{\phi_1 - \phi_3}{2}. \tag{2}$$

For squared momentum transfers, $-t$, where one or more photon exchanges contribute significantly, amplitudes would include an electromagnetic element of the form $(\phi_j + e^{i\delta}\phi_j^e,\ j = 1, \ldots, 5)$, involving a helicity independent[2] Coulomb phase shift[3] $\delta = -\alpha(\ln|bt/2 + 4t/\Lambda^2| - 0.5772)$, where α is the fine structure constant, $\Lambda^2 = 0.71(\mathrm{GeV}/c)^2$ is the dipole Sachs form factor parameter, and b is the nuclear slope parameter. For values of $-t$ corresponding to interference, the amplitude $\phi_4 = \langle + - |\phi| - + \rangle$ may be ignored because of the kinematical factor $(-t)$ for this double helicity-flip amplitude. The electromagnetic amplitudes ϕ_2^e, ϕ_4^e, and ϕ_-^e in the asymmetry

$$A_{\mathrm{N}} = \frac{\mathrm{Im}\{(2\phi_+ + \phi_2)^* \phi_5\}}{|\phi_+|^2 + |\phi_-|^2 + \frac{1}{2}|\phi_2|^2 + 2|\phi_5|^2} \tag{3}$$

are also known to be insignificant at higher energies.[4]

The imaginary parts of the forward hadronic amplitudes ϕ_- and $\phi_2 = \langle + + |\phi| - - \rangle$ are related respectively to the longitudinal and transverse spin-polarized total cross section differences

$$\frac{\mathrm{Im}\,\phi_-(s,0)}{\mathrm{Im}\,\phi_+(s,0)} = \frac{1}{2}\frac{\Delta\sigma_{\mathrm{L}}(s)}{\sigma_{\mathrm{tot}}(s)}, \qquad \Delta\sigma_{\mathrm{L}} = \sigma_{\rightleftarrows} - \sigma_{\rightrightarrows} \tag{4}$$

$$\frac{\mathrm{Im}\,\phi_2(s,0)}{\mathrm{Im}\,\phi_+(s,0)} = -\frac{\Delta\sigma_{\mathrm{T}}(s)}{\sigma_{\mathrm{tot}}(s)}, \qquad \Delta\sigma_{\mathrm{T}} = \sigma_{\uparrow\downarrow} - \sigma_{\uparrow\uparrow} \tag{5}$$

and significant contributions from the absolute values at $t = 0$ of the amplitudes ϕ_- and ϕ_2 would reveal themselves in an enhancement of the forward elastic differential cross section above that expected from estimates involving ρ and the optical theorem. A measure of the enhancement is

$$\beta(s,t) = \sqrt{|\phi_-(s,t)|^2 + \frac{1}{2}|\phi_2(s,t)|^2}\,/|\mathrm{Im}\,\phi_+(s,t)| \tag{6}$$

which in the forward direction may be expressed in terms of the total cross section and total cross section differences and the ratios of real to imaginary parts of the amplitudes ϕ_- and ϕ_2 given by

$$\rho_- = \mathrm{Re}\,\phi_-/\mathrm{Im}\,\phi_-, \qquad \rho_2 = \mathrm{Re}\,\phi_2/\mathrm{Im}\,\phi_2. \tag{7}$$

3 Amplitude ratios

We introduce amplitudes $\psi_+(s,t)$ and $\psi_5(s,t)$, scaled by the imaginary part of the spin-averaged hadronic amplitude, and scaled in addition by a factor of $\sqrt{-t}$, where appropriate

$$\psi_+(s,t) = \frac{\phi_+(s,t)}{\mathrm{Im}\phi_+(s,t)}, \qquad \psi_5(s,t) = \frac{\phi_5(s,t)}{\mathrm{Im}\phi_+(s,t)}\frac{m}{\sqrt{-t}}. \tag{8}$$

If at particular values of s and t the imaginary part here has a zero[5], then an alternative parametrization is required. For values of t that do not include the dip region the expressions given are adequate. We illustrate the partial cancellation of t-dependence for the electromagnetic amplitudes. The interference between $\phi_+^e(s,t)$ and $\phi_+(s,t)$ is most prominent when $t = t_c$ where $t_c = -8\pi\alpha/\sigma_{\rm tot} \approx -0.0018\,(\mathrm{GeV}/c)^2$. Ratios involving the electromagnetic amplitudes, keeping apart a factor t^{-1} due to the photon pole term, when expanded in powers of t yield the following[7]

$$\psi_+^e = -\frac{t_c}{t}\left[1 - \left(\frac{b}{2} - \frac{4}{\Lambda^2} + \frac{\mu-1}{2m^2}\right)t + \cdots\right] \tag{9}$$

including a Taylor series in t for the square of the Dirac electromagnetic form factor $F_1(-t)$ that features in ϕ_+^e, and

$$\psi_5^e = -\left(\frac{\mu-1}{2}\right)\frac{t_c}{t}\left[1 - \left(\frac{b}{2} - \frac{4}{\Lambda^2} + \frac{\mu-2}{4m^2}\right)t + \cdots\right] \tag{10}$$

where a power series in t results from the product of the form factor F_1 and the Pauli form factor F_2 occuring in ϕ_5^e. Here $\mu = 2.793$ is the magnetic moment of the proton, m is the proton mass, and b is typically 12 $(\mathrm{GeV}/c)^{-2}$ near 200 GeV/c.

Introducing ratios with the imaginary-part of the average hadronic helicity non-flip amplitude as denominator

$$\rho \;=\; \mathrm{Re}\,\phi_+/\mathrm{Im}\,\phi_+\,, \tag{11}$$

$$\rho' \;=\; \mathrm{Re}\,(\phi_1+\phi_3+\phi_2)/\mathrm{Im}\,(\phi_1+\phi_3)\,, \tag{12}$$

$$1-\xi \;=\; \mathrm{Im}\,(\phi_1+\phi_3+\phi_2)/\mathrm{Im}\,(\phi_1+\phi_3)\,, \tag{13}$$

we note that $\psi_+^R(s,t) = \rho$, $\psi_+^I(s,t) = 1$, with $\rho' = \rho + \frac{1}{2}\psi_2^R$ and $\xi = -\frac{1}{2}\psi_2^I$ upon writing

$$\psi_j = \psi_j^R + i\psi_j^I\,, \quad (j = 1,\ldots,5,+,-)\,. \tag{14}$$

Because the difference between the parallel and antiparallel transversely polarized proton–proton total cross sections is not expected to be large, the quantitiy ξ will be small by comparison with unity since

$$\xi(s,0) = \frac{\Delta\sigma_{\mathrm{T}}(s)}{2\sigma_{\mathrm{tot}}(s)}, \quad \text{that is,} \quad 1-\xi(s,0) = \frac{\sigma_{\uparrow\uparrow}(s)}{\sigma_{\mathrm{tot}}(s)}. \tag{15}$$

4 Single-spin asymmetry

Since the Coulomb phase shift δ is about 2.5% in the interference region for a proton target with $Z=1$ (and larger for nuclear targets with $Z>1$), its cosine can be approximated by $1-\delta^2/2$ and its sine by δ, and so the asymmetry from Eq. (1–8) reads as follows,

$$\frac{m\,A_N}{2\sqrt{-t}} = \frac{\psi_1^e(\psi_5^I - \psi_5^R\delta) - \psi_5^e(1-\xi-\rho'\delta-\delta^2/2) + \rho'\psi_5^I - (1-\xi)\psi_5^R}{(\psi_1^e)^2 + 2\psi_1^e(\rho+\delta) + \rho^2 + 1 + \beta^2 - 2t\left[(\psi_5^e+\psi_5^R)^2 + (\psi_5^I)^2\right]} \tag{16}$$

The momentum transfer dependence remaining from the approximate cancellation of the the t–dependences arising from the nuclear slope parameter b and the electromagnetic form factors may be written as

$$\psi_+^e = -\frac{t_c}{t} - 0.0025 + O(t) \approx -w - w_e, \tag{17}$$

$$\psi_5^e = -\left(\frac{\mu-1}{2}\right)\frac{t_c}{t} - 0.0010 + O(t) \approx -\left(\frac{\mu-1}{2}\right)w - w_m \tag{18}$$

where $w = t_c/t$. Incorporating $\psi_5^R = R_5$ and $\psi_5^I = I_5$, we have[8]

$$A_N = \frac{\sqrt{-t}}{m} \tag{19}$$
$$\frac{(1-\xi)(\mu-1)w - 2wI_5 + 2\rho' I_5 - 2(1-\xi)R_5}{w^2 - 2(\rho+\delta-w_e)w + 1 + \rho^2 + \beta^2 - \frac{t}{2m^2}\{[(\mu-1)w - 2R_5]^2 + 4I_5^2\}}.$$

The reduced ratios, R_5 and I_5, of the real and imaginary parts of the helicity single-flip to imaginary non-flip amplitude, respectively, are defined as

$$R_5 + iI_5 = \frac{m}{\sqrt{-t}} \times \frac{\phi_5}{\mathrm{Im}\,\phi_+}. \tag{20}$$

The ratio of single-flip to non-flip amplitude is discussed[17] in relation to an estimate of elastic proton-proton polarization at small P_T^2 near 1 TeV. If in the expression for the asymmetry A_{N} above, all quadratic terms in the assumed small quantities ρ, δ, $\Delta\sigma_{\rm T}$, R_5, I_5, w_e are neglected, the asymmetry for the CNI region can be written as follows:

$$\frac{m\,A_N}{2\sqrt{-t}} = \frac{[(1-\xi)(\mu-1)/2 - I]w + (\rho I - R + w_m)}{w^2 - 2(\rho+\delta-w_e)w + 1 + \rho^2 + \beta^2} \tag{21}$$

where $w = t_c/t$, $t_c = -(8\pi\alpha)/\sigma_{\rm tot}$, the transverse spin $\sigma_{\rm T}$ asymmetry is $\xi = \Delta\sigma_{\rm T}/2\sigma_{\rm tot}$, δ is the Coulomb phase, $\mu = 2.7928$ is the magnetic moment of the proton, β^2 is the sum of forward hadronic double spinflip contributions introduced above, and

$$w_e = -(b - 9.23)\,t_c\,/2 = 0.25\%$$

and

$$w_m = -(b - 10.82)\,t_c\,(\mu-1)/4 = 0.10\%$$

are small dimensionless numbers that incorporate the t-dependences of helicity nonflip and flip electromagnetic and hadronic amplitudes to first order in the variable t.

5 Asymmetry maximum

In the expression for the asymmetry $A_{\rm N}$ above, particularly for higher charged Z targets which might be spinless, it can be important to keep all quadratic terms in the small quantities ρ, δ, R_5, I_5, while ignoring terms such as $\Delta\sigma_{\rm T}$, w_e and w_m. Maximizing A_N with respect to t by including $\sqrt{-t}$ written as $\sqrt{-t_c}/\sqrt{w}$ on the right, we find that the maximum of the asymmetry occurs when

$$\frac{t_{\rm max}}{\sqrt{3}t_c} = 1 - (\rho+\delta)\left(\frac{1}{\sqrt{3}} - \frac{32}{3(\mu-1)}R_5\right) \tag{22}$$

$$-\frac{8}{\mu-1}R\left(\frac{1}{\sqrt{3}} - \frac{2}{3(\mu-1)}(5R_5 - \sqrt{3}I_5)\right) - \frac{1}{6}(2\rho^2 - 2\rho\delta - \delta^2) + \ldots$$

in the approximation where the small quantities are kept to second order. Substituting $t_{\rm max}$ in the expression for the asymmetry we find, according to the following equation, that the asymmetry maximum $A_{\rm max}$ receives contributions from the Coulomb phase shift δ, from hadronic ratios ρ, imaginary helicity flip ratio I_5, and the real helicity flip ratio R_5, to second order in such assumed small quantities

$$\frac{4mA_{\rm max}}{\sqrt{-3\,t_{\rm max}}} = (\mu-1)\left(1+\frac{\rho+\delta}{\sqrt{3}}\right)^2 - \frac{2}{\sqrt{3}}(R_5 + \sqrt{3}I_5)$$

$$-\frac{2}{3}R_5(5\rho + 2\delta) - \frac{2}{\sqrt{3}}I_5(\rho + 2\delta) - \frac{16}{3(\mu - 1)}R_5{}^2 + \dots \quad (23)$$

Expected to be about 10% over the RHIC range of energies, a positive value of ρ would enhance the asymmetry maximum in proton elastic scattering. A positive value of either the real or imaginary part of the single helicity flip amplitude, on the other hand, would reduce the asymmetry maximum in proton scattering. Clearly, it would be helpful to know more about the reduced ratios of the helicity flip amplitude, R_5 and I_5.

6 Bound on helicity flip amplitude

Lagrangian optimization techniques may be used to bound the imaginary part of the non helicity-flip amplitude. The imaginary non-flip amplitude may be written, using the optical theorem, as

$$\mathrm{Im}\,\phi_+(s,t)|_{t\sim 0} = \frac{k\sqrt{s}\,\sigma_{\mathrm{tot}}}{4\pi}\,e^{g\,t}\,, \quad (24)$$

where

$$g = \frac{d}{dt}\ln\,\mathrm{Im}\,\phi_+(s,t)|_{t=0} = \frac{1}{\mathrm{Im}\,\phi_+(s,t=0)}\left(\frac{d}{dt}\,\mathrm{Im}\,\phi_+(s,t)\right)\bigg|_{t=0} \quad (25)$$

and in the CNI region, it is sufficient to approximate the exponential term, $e^{g\,t}$, using $(1 + t\,g)$, for the purposes of constraining the helicity-flip amplitude by using a particular $\mathrm{Im}\,\phi_+(s,t)$.

Equality constraints in the Lagrange function include the dimensionless total cross section $k^2\sigma_{tot}/\pi$, the dimensionless elastic cross section $k^2\sigma_{el}/\pi$, and the imaginary non helicity-flip amplitude. The partial wave unitarity relations are written as inequality constraints. The single helicity-flip amplitude, $\mathrm{Im}\,\tilde{\phi}_5$, modified by the a kinematic factor, is introduced as the objective function in the maximization problem.[21]

$$\mathcal{L} = \mathrm{Im}\,\tilde{\phi}_5 + \alpha\left[\frac{k^2\sigma_{tot}}{\pi} - \sum_J (2J+1)\left\{a_0^J + a_1^J + a_{11}^J + a_{22}^J\right\}\right]$$

$$+\beta\left[\frac{k^2\sigma_{el}}{\pi} - \sum_J (2J+1)\left(|f_0^J|^2 + |f_1^J|^2 + |f_{11}^J|^2 + |f_{22}^J|^2 + 2|f_{21}^J|^2\right)\right]$$

$$+\gamma\left[\mathrm{Im}\,\phi_+ - \frac{\sqrt{s}}{4k}\sum_J (2J+1)\left\{a_0^J + a_1^J + a_{11}^J + a_{22}^J\right\}\left(1 + \frac{t}{4k^2}\,J(J+1)\right)\right]$$

$$+\sum_J (2J+1)\lambda_J \left(a_0^J - |f_0^J|^2\right) + \sum_J (2J+1)\nu_J \left(a_{11}^J - |f_{11}^J|^2 - |f_{21}^J|^2\right)$$

$$+\sum_J (2J+1)\mu_J \left(a_1^J - |f_1^J|^2\right) + \sum_J (2J+1)\rho_J \left(a_{22}^J - |f_{22}^J|^2 - |f_{21}^J|^2\right) \quad (26)$$

where α, β and γ are equality multipliers. The inequality multipliers, λ_J, μ_J, ν_J and $\rho_J \geq 0$, should all be non-negative for a maximum.

A detailed study of the interior and boundary unitarity classes indicates that an upper bound on the ratio, $\mathrm{Im}\, r_5 = m_\mathrm{p}\, \mathrm{Im}\, \phi_5/(k\, \mathrm{Im}\, \phi_+)$, is given by

$$\mathrm{Im}\, r_5 = m_\mathrm{p}\sqrt{g}\left(\frac{36\pi g \sigma_{el}}{{\sigma_{tot}}^2} - 1\right)^{1/2} \times f(t)\,. \quad (27)$$

The function,

$$f(t) = \frac{\left(1 + 2tg + 9/8t^2g^2\right)^{1/2}}{1 + tg}\,, \quad (28)$$

is unity at zero momentum transfer and in the Coulomb Nuclear Interference region, $f(t) \sim 1$. The upper bound on $\mathrm{Im}\, r_5$ with centre of mass energies in the RHIC range, 50 to 500 GeV, is less than the critical value of $\kappa_p/2 = 0.896$. We conclude that the asymmetry, A_N, in the CNI region is non-zero and positive for proton proton collisions.

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